PROBLEM SET #1

Galactic Structure and Blackbodies; 40 pts total

due Thursday, 2/13/2014

1. Stellar cluster problem [26 pts]

The following refers to a star cluster observed on Aug 15, 2010 at about 4am UT. The cluster is located near the border of Cygnus and Cassiopeia at RA (α) = 21^h 58_m 15^s and Dec (δ) = $54°$ $48'$ $33''$. There are at least several hundred stars in the cluster, and the mean radial angular distance from the center of the cluster is 50 arcminutes.

(a) [2 pts] Use Fig. 24.18 in BOB to convert the cluster's equatorial coordinates (α,δ) to galactic coordinates (l,b). Draw a picture of the galaxy as observed from the north galactic pole. Indicate the galactic center, the location of the Sun, the orbital direction the Sun is moving, and the observation direction to the cluster.

(b) [2 pts] One of the more massive stars in the cluster appears to have a spectral class of A5 and looks like it is on the main-sequence. The absolute V- magnitude and color index for such a star are M_V = 2.2 and B−V = 0.14, respectively. The observed magnitude is V = 9.8, and the observed color is B−V = 0.25. Estimate a distance to the cluster assuming a standard reddening law, $A_v = RE_{B-V}$ with $R = 3.1$.

(c) [2 pts] Given the distance to the cluster, calculate the radial velocity of the source if it moves with the general galactic rotation. Argue that the object should be either blueshifted or redshifted based on your diagram in part (a), and how the galactic rotation curve behaves.

(d) [2 pts] Find the ecliptic coordinates (λ, β) of the cluster from the formulae:

$$\sin\beta = sin\delta\cos\epsilon - \cos\delta\sin\alpha\sin\epsilon \tag{1}$$

and

$$\cos\lambda = \frac{\cos\delta cos\alpha}{\cos\beta} \tag{2}$$

where $\epsilon = 23.439$ degrees is the tilt of the Earth's axis to its orbital plane. Look at your answers to make sure you got the signs on all the arcsines and arc-cosines correct. It will help to look at a globe of the celestial sphere to check your answers.

(e) [3 pts] Draw a picture of the Earth's orbit as seen from the north. Put the vernal equinox to the right. Place the Earth on its orbit at the time of the observations and indicate its position to within a degree or so. Put an arrow on the orbit to indicate the direction of the Earth's orbital motion. In what direction in ecliptic coordinates (λ, β) is the Earth moving at this time? Draw an arrow in this direction from the Earth and another in the direction of the cluster. How far out of the ecliptic plane is the cluster in degrees? Would you expect the cluster to have an extra blueshift or redshift from terrestrial motion based on the diagram? Given that the Earth's orbital motion is 30 km/s, what should that blueshift or redshift be (fairly simple trig here)? Note we are ignoring the Earth's rotation ($\sim$ 0.1 km/s), and the pull of the Moon ($\sim$ 10 m/s) for this problem.

(f) [2 pts] The Sun has a peculiar velocity relative to the local standard of rest of about 14 km/s, and it moves in the direction of the constellation of Hercules, at approximately 18h +30. Would you expect this motion to cause an extra blueshift or redshift to the cluster? Estimate the amount of this shift. Don't worry too much about getting the angle between the solar apex and the cluster exactly right, but try to get it to 5 degrees or so.

(g) [2 pts] Combine the results from parts (a) through (e) as follows. Assuming that the cluster has no peculiar motions (i.e. rotates with the general galactic motion), what would you expect the observed redshift or blueshift to be, taking into account (i) the Earth's motion around the Sun,

(ii) the Sun's motion relative to the LSR, and (iii) the galactic rotation curve?

(h) [2 pts] The following data were taken for the location of the center of the Hα absorption line for the brightest 25 stars in the cluster. The rest wavelength in the lab for Hα is 6562.80Å.

6561.804	6561.797	6561.790	6561.795	6561.795
6561.794	6561.797	6561.789	6561.807	6561.804
6561.797	6561.799	6561.796	6561.823	6561.802
6561.798	6561.785	6561.794	6561.805	6561.800
6561.800	6561.807	6561.800	6561.798	6561.805

Calculate the observed radial velocity and peculiar radial velocity relative to the LSR of the cluster in km/s. In practice it would be difficult, though not impossible, to measure the wavelengths to this level of precision.

(i) [3 pts] The cluster appears to have $\sim$ 10 members to it that have masses greater than 1 $M_\odot$. Assume a Salpeter initial mass function describes the cluster's mass distribution down to 0.1 $M_\odot$. That is, N(m)dm $\sim m^p$, where N(m)dm is the number of stars with mass between m and m + dm, m is the mass of the star, and p = -2.35. How many stars should be in the cluster if that is the case? What should the total cluster mass be? What is the most likely spectral type of the most massive star (cf. Appendix G in BOB)?

(j) [2 pts] Estimate the mass of the cluster in solar masses using the radial velocity data. Compare the results with the estimate in part (i).

(k) [1 pt] Estimate a typical orbital time for a star in the cluster in years. Recall that the timescale for all graviational phenomena is $(G\rho)^{-1/2}$, where ρ is the mass density.

(l) [2 pts] Observations show that the cluster stars have a common proper motion of 5 milliarcseconds per year. The proper motion vectors intersect at a point on the sky 12 degrees away from the center of the cluster. Use this information to recalculate the distance to the cluster, and compare the result with what you got in part (b). Should the proper motion vectors be converging or diverging? Why?

(m) [1 pt] Estimate the average distance between the stars in the cluster.

2. Blackbody Practice [14 pts]

In the following take $F_\lambda = \pi B_\lambda$, where B_λ is the Planck function. Consider the function

$$\lambda F_\lambda = \frac{2\pi hc^2}{\lambda^4} \frac{1}{e^{hc/\lambda kT} - 1}$$

Note that λF_λ has units of *flux*, not specific flux.

(a) Find the wavelength, λ_m, at which λF_λ is a maximum, in terms of the temperature.

(b) Find the total flux F_{tot} in terms of the maximum value of λF_λ.

(c) Suppose a star emits like a blackbody. Find the angular diameter θ in arcseconds in terms of the temperature of the star and the observed value of $(\lambda F_\lambda)_{max}$.

(d) The following magnitudes have been observed for the bright red star Antares (α Sco), and for Mars:

Band	α Sco	Mars
B	+2.79	+1.03
V	+0.96	−0.34
R_C	−0.10	−1.02
I_C	−1.33	−1.61
J	−2.85	−2.13
H	−3.60	−2.42
K	−3.87	−2.47
L	−4.10	−2.21
M	−3.92	−4.24
N	−4.50	−9.55
Q	−4.87	−11.51

Plot these data on one of the sheets provided.

(e) Find the temperature and angular diameter of α Sco from the above data by fitting a blackbody to the observations (the 2000K blackbody curve will be useful for this). Plot the observations of α Sco along with the fit. Comment on any deviations from a blackbody that you find.

(f) The distance to α Sco is 130 pc. What is the radius of the star in AU? If the Sun were to someday evolve into a star like α Sco, what would the Sun look like from the Earth?

(g) Explain any similarities or differences of the Mars data from the α Sco data.