PROBLEM SET #3

Cosmology; 26 pts total

due Friday April 25, 2014

1. [3 pts] Prove or give a counterexample: Universes that have identical a(t) for all time t must have the same geometry.

2. [10 pts] Suppose the microwave background radiation were four times hotter than it is actually observed to be. We will let Ω_b, Ω_Λ, and Ω_m be the same in this new 'hot' universe.

(a) [1 pt] Calculate a new n_γ.

(b) [1 pt] Calculate a new η.

(c) [2 pts] Estimate a new recombination temperature T_{REC}. Explain why you would expect the temperature to be higher or lower than in the actual case.

(d) [1 pt] Calculate the redshift of recombination z_{REC}.

(e) [3 pts] Construct a table of T and z for the following values of X in both the present universe and this new one. X = 0.001 - 0.009 increment 0.001; 0.01 - 0.09 increment 0.01; and 0.1 - 0.9 increment 0.1. What is the value of the redshift z_{DEC} and temperature T_{DEC} at decoupling? You may ignore non-equilibrium ionization effects.

(f) [1 pt] Does nucleosynthesis change in any significant manner?

(g) [1 pt] What would you expect for a maximum angular fluctuation size relative to what we currently see?

3. [2 pts] From class notes, the pressure P of an isotropic gas that contains particles of momentum p is given by $P = nvp/3$. Calculate an expression

for the equation of state parameter w as a function of the velocity v that is valid for all velocities. What does w approach in the highly-relativistic and non-relativistic limits?

4. [5 pts] The current Universe is described well by a mixture of lambda and matter. In this problem you will estimate the 'effective' current w-parameter that is a mixture of these quantites.

[1 pt] (a) First, derive a general relationship between a(t), $H_\circ$ and (t-$t_\circ$) that holds for any a(t) in the limit that (t-$t_\circ$)/$t_\circ$ << 1.

[2 pts] (b) For a single component w-universe we found the scale factor to be $a(t) = a(t_\circ)[t/t_\circ]^p$ where p is some power. Expand this relationship to first order in (t-$t_\circ$)/$t_\circ$, and match the coefficients to those in part (a). This gives you an equation for the effective power law p in operation at any given time $t_\circ$ for an arbitrary a(t). Note that p is a function of $t_\circ$.

[1 pt] (c) Use the equations on page 90 of Ryden for the flat lambda-matter universe to determine the current effective value of the w-parameter. Use $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$.

[1 pt] (d) Solve for the effective value of w at some earlier time when $\Omega_m = 0.5$ and $\Omega_\Lambda = 0.5$.

5. [2 pts] We observe galaxies at a variety of redshifts. Let d_e denote the proper distance that the galaxy was at in the comoving frame when the light was emitted. Derive the maximum distance $d_e(z)$ for a matter-dominated universe.

6. [4 pts] Ryden Problem 11.1: Planck Omegas