Model Stellar Atmosphere

due Friday, 12/9/2016

50 pts

In this exercise you will generate a model atmosphere for a late-G/early-K main-sequence star. Use the following values: $T_{e\star} = 5340$ K, $R_\star = 0.79\ R_\odot$, $M_\star = 0.78\ M_\odot$. Please use cgs units throughout the problem for ρ and P. I would like everyone to do their own coding and come up with their own tabular values. This is not a group homework assignment.

Order of Magnitude Estimates

Before you start with serious modeling you need to know ball-park estimates for g, P, P_e, n, ρ, and T that apply to the stellar atmosphere. You are to calculate these as indicated below, not look them up somewhere. A calculator is all you will need for this part.

(a) For the mass and radius given above, determine log $g_\star$ in cgs units.

(b) Estimate the pressure P at $\tau = 1$ in terms of the opacity κ. Use the Table of opacities handed out in class to find approximate values for ρ, κ, and P in the atmosphere. How does the density of the stellar atmosphere compare with that of the air you are now breathing? If your initial guess for κ looks way off you may want to loop once or twice by hand.

(c) Use $T \sim T_e(\star)$ to estimate the electron pressure P_e.

Getting Started

With order of magnitude estimates in hand you now have some idea as to where you will be looking to interpolate in the opacity tables. Your goal is to calculate the temperature, density, pressure, and distance with optical depth. The model should have 30 values for τ : 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9 and 2.0.

(d) For these values of τ use the Eddington approximation to calculate $T(\tau)$, given that the observed effective temperature of the star is 5340 K. Print out a Table 1, which should have τ in the first column, $T(\tau)$ in the second column, and $\theta = 5040/T$ in the third column.

(e) For the range of θ applicable to the model, we will need to know how the opacity varies with P. The graph handed out in class shows the Rossland mean opacity over the parameter space of interest. It looks as if the behavior of κ with P is mostly linear, with a slight curve, indicating it is probably well-matched by a power law.

We will match the values in the graph with a fit according to a law

$$\kappa = a(\theta)P^{b(\theta)} \tag{1}$$

Ideally we might fit a power law to each curve in the graph using several points for the fit. This would take a long time. Instead, we will pick two values for P and fit a power law to those points for a few curves, and then interpolate between curves. The values $P_1 = 6 \times 10^4$ and $P_2 = 12 \times 10^4$ are widely separated enough to ensure the fit will be reasonable over the entire range, but not so widely separated that the fit will fail in between.

For these two points, construct Table 2: columns of θ, $\kappa(P_1)$, $\kappa(P_2)$, b(θ), and a(θ) for the following values of θ: 1.1, 1.05, 1.00, 0.95, 0.90, 0.85, 0.80, 0.77, 0.75, and 0.70. The values of $\kappa(\mathrm{P}_1)$ and $\kappa(\mathrm{P}_2)$ for $\theta = 0.70$ are off your graph, and are 1.442 and 1.962, respectively.

(f) To get an estimate of P(τ), we can use

$$\frac{dP}{d\tau} = \frac{g}{\kappa} \tag{2}$$

by using a power law approximation for κ with P, ignoring for now the temperature dependence of κ. So pick a value of θ somewhere in the middle of Table 1, and use the power law fit you found in Table 2 to solve for P(τ). Check that your answer is consistent with the estimate you did in part (b).

(g) Use solar abundances (X = 0.71, Y = 0.27, Z=0.02) for the star. Find the mean molecular weight and use the ideal gas law to determine the density $\rho(\tau)$. Remember that the gas is mostly neutral, so you can't just use the formula for the interior which assumed full ionization.

Construct Table 3, which is Table 1 with two new columns added - the fourth column is P in units of 10^4 dyne cm^{-2}, and the fifth column is the mass density ρ in units of 10^{-8} g cm^{-3}.

Let the Loop Begin!

The preceding solution did not take into account the change of opacity with temperature that you found in Table 2. There is some advantage to starting in the middle and working each way to the boundaries, but for our purposes we will simply start from the top ($\tau = 0.05$) and work down to the bottom ($\tau = 2$).

(h) Write a code that uses your initial solution for T(τ) and P(τ) in Table 3 and your fits to the opacity in Table 2 to find the opacity $\kappa(\tau)$ at each point in your grid. Describe clearly how you did the interpolation.

Run your program until the solution converges. Print out the following for each iteration of the grid point at $\tau = 0.5$ and put it in Table 4:

column 1: iteration number
column 2: P in units 10^4 dyne cm^{-2}
column 3: ρ in units 10^{-8} cm^{-3},
column 4: κ

Are we there yet?

(i) Well, we haven't calculated the distance r(τ). We may as well take r to be 0 at our first point, τ=0.05. Use the equation of hydrostatic equilibrium to find a distance for each grid point and add this feature to your program.

Finally, print out your solution as Table 5:

column 1: τ
column 2: T (K)
column 3: θ
column 4: P in units 10^4 dyne cm^{-2}
column 5: ρ in units 10^{-8} cm^{-3},
column 6: κ
column 7: r (km)

and compare it to the one you made in Table 3.

(j) One thing more to check is to make sure that the gas is mostly neutral everywhere. Use the Saha equation at $\tau = 0.1$ and $\tau = 2$ to calculate the ratio P_e/P ($\sim n_e/n_H$)

(k) Now that you have your model, let's use it to predict some observable quantities. Because you used a table of Rossland mean opacities, you cannot calculate any absorption lines with your code directly. However, you can use it to predict limb darkening. To this end, first calculate the brightness temperature in the V-band (5500Å) and μ=1.0 from your model by integrating the source function over all depths. You will need to write a new code (call it 'bt' for brightness temperature). Calculate the brightness temperature for μ = 0.1, 0.2, ... 1.0, and compute the ratio of the observed flux to that at $\mu = 1$. Then redo this calculation again for the K-band, a wavelength of 2.2 μm. [Suggestion: You are going to need T(τ), so you may as well do as you did for part d, but now you must go to much greater optical depths in order to make sure you don't miss radiation coming from large τ. However, at small μ, most of the action occurs at very low τ so your grid there needs to be finer than 0.05. You might experiment with different grids. Clarify what final grid you used in your writeup].

Your star is not too different from the Sun, so maybe it looks similar. You can compare your results to analytic fits to the observed solar values (e.g. 1977, Solar Phys. 51, 45; 1977, Solar Phys. 52, 179) with the formula

$$I(\mu)/I(0) = 1. + a_1 x + a_2 x^2 \tag{3}$$

where x = ln(μ). For the Sun, a_1 = 0.437 and a_2 = 0.073 at 0.55μm, and a_1 = 0.145 and a_2 = 0.015 at 2.2μm. As a sanity check, there is a graph of limb darkening for various wavelengths in Chapter 9 of Gray.

Collect your results in Table 6, which should have a format like

TABLE 6
Wavelength = 0.55 microns
Tb = 5940.9; μ = 1.0; I(μ)/I(0) = 1.00; solar = 1.00
Tb = 5998.3; μ = 0.9; I(μ)/I(0) = 0.93; solar = 0.87
...
Wavelength = 2.20 microns
Tb = 5920.4; μ = 1.0; I(μ)/I(0) = 1.00; solar = 1.00
Tb = 5783.6; μ = 0.9; I(μ)/I(0) = 0.98; solar = 0.99
...

How close is your model to the solar case?

(l) Include a copy of the codes you used to do this assignment

Final Comments

Recall we assumed an Eddington law for T(τ). A better solution, one that does the radiation balance correctly, is the q(τ) formalism of discrete ordinates that you did for homework #4. This correction amounts to about 4% in T at $\tau = 0$, and less than 1% for $\tau > 1$. Hence, this is not a huge factor, but is one that is easy enough to correct for by simply using the q(τ) formula for T(τ) instead of Eddington. The next level of improvement would be to introduce a non-grey atmosphere and employ one of the correction schemes discussed in Chapter 12 of Collins. Then we might try putting in all the atomic physics and see what sort of line strengths and line shapes we get out. But this is more detail than we can go into with only a half semester to study stellar atmospheres!