**PROBLEM SET #2, ASTR 451/551**

Polytropes, Virial Theorem, Macrostates and Microstates, Homology

[24 pts] due Wednesday, September 28, 2016

1. Microstates and Macrostates for Maxwell-Boltzman, Fermi-Dirac and Bose-Einsten [6 pts]

Chapter 1, problem 2 of Collins. Also calculate $W_{333}/W_{TOT}$ for MB, FD and BE.

2. Practice with observational data [8 pts]

(a) [1 pt] You observe a star with a spectrum that appears to be somewhat hotter than the Sun. Looking at it a bit more closely, it looks like an F0V star. The star has an observed visual magnitude $m_V = 14.0$. The luminosity of the star ought to be 5.21 $L_\odot$ if it is on the main sequence, the mass 1.6 $M_\odot$, and the color index of an unreddened F0V star is $(B-V)_\odot = 0.30$. The Sun's apparent visual magnitude is $-26.7$. How far away is the star (in parsecs [pc]) assuming there is no dust along the line of sight and ignoring for now bolometric corrections?

(b) [1 pt] In part (a) you ignored bolometric correction terms, and yet you still got the right answer. If you look up bolometric correction tables for V-magnitude, you'll see that those for the Sun are similar to those for an F0V star, while those for O and M stars are much more negative. Explain this behavior, including the minus signs.

(b) [1 pt] Someone later observes that the color of the star is $B-V = 1.10$. Recalculate the distance assuming a standard extinction law with $R = 3.1$.

(c) [2 pts] That star appears to have an exoplanet! The maximum eclipse depth (flat-bottomed shape) is 0.007 magnitude, and the period is 18.6 hours. Calculate the orbital semimajor axis in AU and the radius of the exoplanet. Estimate the radius of the star assuming it is a blackbody with a temperature 7300K (the solar temperature is 5770K). Draw the system to scale.

(d) [2 pt] Is it worthwhile to write an observing proposal to follow the radial velocity of the star? What should the amplitude of the radial motions be in km/s if the planet is a gas giant with an average density of $\rho \sim 1$ g/cc? A good spectrograph may be able to centroid the radial motions of stellar absorption lines up to a factor of about 50 times the resolution $R = \lambda/\Delta\lambda$ of the spectrograph. The maximum resolution you can get without special equipment and still get reasonable signal-to-noise in 30-minute integration for a 14th magnitude star on a large telescope is about 30000. With special equipment like gas cells one can get down to $\sim$ 10 m/s if the stellar photosphere is very stable.

(e) [1 pt] Assume that somehow you have an instrument that reduces the glare from the star enough that you could try to image the planet directly. How close would the system have to be for you to separate the light from the planet from that of the star (1" resolution set by seeing) or from the Hubble Space Telescope (resolution $\sim \lambda/D$, where D = 2.3-m and $\lambda$ is in the optical)?

3. Virial Theorem for Isothermal Cores [6 pts]

Consider a star that has an isothermal core of mass $M_C$, radius $r_C$, temperature $T_C$ and mean molecular weight $\mu_C$. An envelope of mass $M_{ENV}$ surrounds the core, with $M = M_C + M_{ENV}$ being the total mass of the star. The envelope exerts a pressure $P_{surf}$ at the surface of the core. You may assume the ideal gas law.

(a) [1 pt] Apply the general virial theorem to the core, and solve for $P_{surf}$ in terms of $r_C$, $M_C$, and $T_C$.

(b) [1 pt] Sketch $P_{surf}$ vs. $r_C$. Note the asymptotic behavior of the solution for large $r_C$ and small $r_C$. Describe in words what you can infer physically about the star from the shape of the curve.

(c) [1 pt] For a fixed value of $P_{surf}$ there may be multiple solutions for $r_C$. Which ones (if any) are stable, and why?

(d) [2 pts] Solve for the radius that supports the maximum value of $P_{surf}$ in terms of $M_C$ and $T_C$. Solve for the maximum value of $P_{surf}$ as a function of $M_C$ and $r_C$.

(e) [1 pt] The isothermal core solution must be matched to one for the rest of the star at $r_C$. In particular, we need to have $P(r_C) = P_{surf}$. To do this rigorously requires a fair bit of work to match the polytropes. Instead, we will obtain an approximate expression for $P(r_C)$ by noting $P = F/A$, where A is the area of the core at $r_C$, and F is the gravitational force on the core from the envelope. For point masses, $F = GM_C M_{ENV}/d^2$, where d is the distance between the point masses. Here, the envelope begins at $d = r_C$ and extends to larger r. As an estimate, let's take $d = 2r_C$ to account for the fact that while the densest part of the envelope will be at $r_C$, some material that comprises $M_{ENV}$ is at larger distances.

Equate the gravitational pressure of the envelope at $r_C$ to $P_{surf}$, and solve for $M_C/M$.

4. Homology Invarients [4 pts]

[2 pts] (a) If $\theta(\xi)$ is a solution to the Lane-Emden equation, determine an expression for x such that $\theta'(\xi') = A^x\theta(A\xi)$ is also a solution. $\theta'(\xi')$ is known as a homology solution to the equations.

[2 pts] (b) Two polytropic stars (denoted with the subscripts 1 and 2), are homologous provided that if $r_1 = C_1 r_2$ then $M_1(r_1) = C_2 M_2(r_2)$, where $C_1$ and $C_2$ are constants. Define

$$v = -\frac{d \ln P}{d \ln r}. \tag{1}$$

Find the simplest possible general relationship between $v_1$ and $v_2$.