

PROBLEM SET #2, ASTR 451/551

Stellar Evolution from Cradle to Grave

[28 pts] due Friday, November 4, 2016

1. PMS Radiative Tracks and the Main Sequence Locus in the HR Diagram [12 pts]

(a) [2 pts] The class handout showed some pre-main-sequence (PMS) tracks for various masses, and a solid line depicting the zero-age main-sequence (ZAMS). Draw a line on the radiative track for $1.0 M_{\odot}$ and use the slope to determine a typical power law $L \sim T_e^{Y_{rad}}$ for the radiative portion of the tracks, and $L \sim T^{Y_{ms}}$ for the ZAMS, where Y_{rad} and Y_{ms} are constants. Include a copy of the plot that shows how you measured the slope. Note that Y_{rad} refers to a situation where the mass is constant, but this is not true for Y_{ms} .

(b) [2 pts] Take the opacity law to be

$$\kappa = \kappa_0 \rho^{\alpha} T^{-\beta} \quad (1)$$

Use relationships of variables in stellar interiors to solve for X and Y, where

$$L \sim M^X T^Y \quad (2)$$

Here, T is a typical interior temperature for the star.

(c) [2 pts] A common analytical opacity law is Kramer's opacity, where $\alpha = 1$ and $\beta = 7/2$, while a typical fit to numerical calculations gives somewhat different values of $\alpha = 0.5$ and $\beta = 5/2$ over the internal density and temperature ranges of interest. For these two cases, determine the slope of a pre-main-sequence track in the HR diagram for a radiative track. Which is closer to what you measured in (a)?

(d) [1 pt] What relationship of M to L would you recover if you assumed, like we did in class, that $\kappa = \text{constant}$?

(e) [2 pts] Use your measured value of the slope of the main sequence with both Kramer's opacity and the numerical value to derive a relationship between M and L on the main-sequence. Which one is more in line with the observations we discussed on the first day of class?

(f) [2 pts] In the above, we had to measure the slope in the HR diagram to derive a mass-luminosity relationship. It would be nice if we could just do all of this theoretically. The missing piece of physics is that we have not put fusion into this. Use the numerical value of κ together with the temperature dependence T^ν for the p-p chain reactions to eliminate Y and solve directly for the X defined in part (b).

(g) [1 pt] Does the value of X you found in (f) look right? Explain why the estimate in part (f) may be high, low, or accurate as it is.

2. Equations of Stellar Interiors [3 pts]

In class we derived

$$\begin{aligned}\frac{dM(r)}{dr} &= 4\pi r^2 \rho(r) \\ \frac{dP(r)}{dr} &= -\frac{\rho(r)GM(r)}{r^2} \\ P(r) &= \frac{\rho(r)kT(r)}{\mu m} \\ \frac{dT(r)}{dr} &= -\frac{3\alpha L(r)}{64\pi\sigma T^3(r)r^2} \\ \frac{dL(r)}{dr} &= 4\pi r^2 \rho(r)\epsilon(r)\end{aligned}$$

where $\epsilon(r)$ is the energy generation rate from fusion and is a function of P(r) and T(r). So, there are 5 equations, 5 unknowns

[$T(r)$, $P(r)$, $M(r)$, $L(r)$, $\rho(r)$], and two ‘parameters’, $\epsilon(r)$ and $\mu(r)$ that can be followed with time.

Why didn’t we simply take

$$L(r) = 4\pi r^2 \sigma T^4(r) \quad (3)$$

as our fourth equation, and get rid of the dT/dr equation? Is there something wrong with that equation? It sure looks simpler to deal with.

3. Hot-Bottom Burning in Stellar Evolution [3 pts]

Explain what hot-bottom burning is, including what phase of stellar evolution it refers to, how it comes about, and how it changes the observed surface abundances of the elements. You may wish to make use of the website

<http://sparky.rice.edu/~hartigan/astr551/movies/>

4. Accretion Disks around Compact Objects and Young Stars [10 pts]

Compact objects and T Tauri stars are sometimes surrounded by optically thick disks of gas and/or dust that radiate as a blackbody to a good approximation at optical and near-infrared wavelengths. Consider a geometrically flat disk with a temperature dependence $T \sim r^{-\alpha}$, where α is a constant. Suppose you observe the disk face-on.

(a) [3 pts] Write a general expression for the luminosity per unit frequency L_ν for this temperature law. You may assume the disk has some inner radius r_1 and outer radius r_2 . Note that $\nu L_\nu = \lambda L_\lambda$. The observed spectrum at some distance will have the same shape as L_ν when plotted against ν , or as L_λ when plotted against λ .

(b) [3 pts] Rewrite the integral in part (a), grouping together any constants that do not depend on the frequency. You will

want to write the integral in dimensionless form so that all the dependencies on frequency can be brought outside the integral. The integral then has bounds from x_1 to x_2 , where x_1 and x_2 are dimensionless. Assuming $x_1 \ll 1$ and $x_2 \gg 1$, these bounds approach 0 and ∞ , respectively, and the integral becomes a fixed number, which you can simply group along with the other constants. You are only interested in the frequency dependence. Show that $L_\nu \sim \nu^\beta$. Derive a relationship between β and α .

[2 pts] Note that this power-law relationship only holds if your integral goes from 0 to ∞ . Define the frequency range over which the power law should hold. Explain why it fails at the high-frequency and low-frequency ends. It might help you to draw a picture of the contributions of several annuli to the total spectrum, beginning at r_1 and ending at r_2 . As usual, accurately-drawn diagrams will be more beneficial to you.

(c) [2 pts] What should the temperature law be if the disk simply absorbs light from the star and reradiates this light as a black-body? Input this law into your expression in (b) to determine the spectral energy distribution λL_λ of infrared excess emission for a typical T Tauri star.