ASTR 542
PROBLEM SET #1
Physics of Photoionization, Recombination, Charge Exchange & Radiative Transfer

due Monday Feb 20, 2017

1. The Milne relation allows us to obtain recombination cross sections provided we know what the photoionization cross sections look like. Calculating the photoionization cross sections takes some work, because it liberates a free electron, but one that still lives within its original Coulomb potential. Fortunately, most of the relevant photoionization cross sections follow power laws.

Assume the photoionization cross section

$$a_\nu = \mathrm{n} a_{\nu_\circ} \left(\frac{\nu}{\nu_\mathrm{n}}\right)^{-\mathrm{s}} \tag{1}$$

where $\nu_n = \nu_\circ/\mathrm{n}^2$, with $\nu_\circ$ the threshold frequency for ionization from the ground state, s is a power law index, and n is the usual electronic quantum number. Use the Maxwellian probability distribution for v:

$$f(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} \tag{2}$$

to find an equation for the rate coefficient

$$\langle \sigma_{fb} v \rangle \tag{3}$$

for recombination (free-bound). You should use the symbol

$E_n(z)$ for the exponential integral:

$$E_n(z) = z^{n-1} \int_z^\infty \frac{e^{-x}}{x^n} dx = \int_1^\infty \frac{e^{-zx}}{x^n} dx \qquad (4)$$

to help display your answer.

Evaluate the constants in the expression and explicitly point out temperature dependencies in the limits of $h\nu_n << kT$ and $h\nu_n >> kT$ for both s = 2 and s = 3. For hydrogen, you may take $a_{\nu\circ} = 6.3 \times 10^{-18}$ cm^2.

Recombination is a way for the gas to cool. The energy lost by the electron when it recombines goes into an emitted photon, which leaves the system (or is scattered around). From the shape of the *photoionization* cross sections we have now obtained expressions for how the *cooling rate by recombination varies with temperature.* This is an important connection between the atomic physics and a cooling process that helps determine a nebula's temperature.

2. A pure hydrogen nebula has a density 10^4 cm^{-3}, a temperature 10^4 K, and is located 1 pc away from a star that has a blackbody spectrum in the UV, with $T_* = 5 \times 10^4$ K and $R_* = 10\ R_\odot$. You may assume Case B recombination applies and use the OTS approximation for Lyman continuum photons.

Visualize a small cube, 1 cm^{-3} in size, with $N_e = N_{H_{II}} = 10^4$ cm^{-3}, and the neutral fraction of the gas is about 10^{-4}. Calculate the following timescales, in days (or years, if that makes more sense), for the cube:

(a) The average time between recombinations from the continuum to levels 1, 2, 3, and 10.

(b) The average time for a photoionization from level 1.

(c) The average time for a photoexcitation from level 1 to 2.

3. Consider a homogeneous, spherical gas cloud which is characterized by an emission coefficient j_ν and a source function S_ν. The cloud has a radius R, and the optical depth along its diameter is τ_ν.

(a) What is the flux F_ν at a distance $r > R$?

(b) Plot the flux as a function of τ. What does your expression reduce to in the optically thin limit? Does the answer make sense?

4. Determine $[N^+]/[N]$ as a function of $[H^+]/[H]$ and the temperature T if charge exchange processes dominate the ionization balance of N. Plot your results as a function of $\log(T) = 3$ to $\log(T) = 4.5$. You may ignore energy levels above 4eV when doing this problem.

5. [DUE Friday March 10 with HMWK 2] Write a program that calculates the populations in a 5-level atom, given all the A_{ul} for u>l and l=1-4, and all the Ω_{ij} collision strengths. You will also need to put in all the energy levels. Now look up (record and reference) all the values for the lowest 5 levels of S II. Your

code should predict the emission line ratios you obtain for any input temperature T and electron density n_e. You observe the following line ratios:

[S II] λ6716 / [S II] λ6731 = 0.644;
[S II] $\lambda\lambda$(4068+4076) / [S II] $\lambda\lambda$(6716+6731) = 0.143.

Estimate the temperature and electron density. Unfortunately, the λ4068 line is blended with the λ4076 line. If you could deblend them, perhaps with better observations, what would you predict for their ratio?