ASTR 542: PROBLEM SET #3
Emission Lines, Grains, Shocks & Molecules
Model Strømgren Spheres
due Monday May 1, 2017

1. [5 pts] An atom has a ground state (level 1), and two excited states (levels 2 and 3). Transition 3 – 1 is permitted (call it P) and 2 – 1 is forbidden (call it F). Levels 2 and 3 are roughly the same energy above level 1, and have identical statistical weights and $\Omega_{12} = \Omega_{13}$. The ratio of Einstein A's for the two transitions is 10^8. Both levels are populated by collisions. The nebular temperature is constant. Plot the log of the flux observed for both lines *vs.* the log of the density N in the gas. Make sure to include any optical depth effects. Explain physically what is going on in your plot and carefully consider how the lines behave given their A-values.

2. [5 pts] Use the jump conditions for a non-magnetic shock to make a rough sketch of the density and temperature as a function of distance behind

(a) A Mach 10 nonradiative shock,

(b) A Mach 3 nonradiative shock,

(c) A Mach 10 radiative (isothermal) shock,

and (d) A Mach 10 radiative (isothermal) shock where $B \neq 0$.

3. [10 pts] A dust grain of radius $a = 0.2\mu$m sits in an H II region that has a density 10^3 cm^{-3}. The grain has a charge Ze, where e $= 4.8 \times 10^{-10}$ esu is the charge on an electron. Assume that all particles hitting the grain stick to it.

(a) Calculate the cross section of the grain to electron impact as a function of the velocity v of the electron. Do the same for a proton.

(b) Do a rough estimate of Z for the grain. You can do the integrals if you want, but for the purposes of this problem you can also simply substitute the average velocity of protons and electrons into your formulas for the cross sections. How long does it take for the grain to charge up?

4. [5 pts] Determine an expression for the most populated J level in the $v = 0$ vibrational state of CO as a function of T. If the temperature of a typical molecular cloud is 20 K, what is the most populated level? You will need to look up the moment of inertia for this molecule somewhere to do this problem.

5. [25 pts] Begin with the ionization equation, and use the on-the-spot approximation to show that the degree of ionization x of a pure hydrogen gas surrounding a star that radiates like a blackbody can be written as

$$\frac{x^2}{1-x} = \frac{2\pi}{c^2}\frac{R_*^2}{r^2}\frac{\int_{\nu_1}^{\infty} \nu^2 \left(e^{h\nu/kT_*} - 1\right)^{-1} e^{-\tau_\nu} a_1(\nu) d\nu}{N\alpha_B}$$

where τ_ν = N $a_\nu \int_{R_*}^{r} (1-x)\, dr$, and all other symbols are as defined in class.

We wish to solve this equation for x. Because of the x-dependence of τ, this equation cannot be solved analytically. To solve numerically, break the calculation into a number of shells, each of width Δr, where Δr is chosen to be some suitably small (e.g. 0.01) fraction of the size of the Strømgren sphere found from the simple calculation done in class.

Begin with $r_1 = R_*$, where $\tau = 0$. The integral should be done using Gaussian-Leguerre quadrature, and you can then solve the quadratic equation for x_1. Assume that this ionization holds within the first shell. To find x_2 you need to know $\tau_\nu(r_2)$, but this is simply $Na_\nu(1-x_1)\Delta r$. Use this value to solve the equation again for x_2.

Solve this problem for the case where $N = 1\ cm^{-3}$, $\alpha_B = 2.6 \times 10^{-13}\ cm^3\ s^{-1}$, and $T_e = 10^4 K$. Take the absorption cross section to be $a_\nu = 6.3 \times 10^{-18} \left(\frac{\nu_1}{\nu}\right)^3\ cm^2$, $T_* = 40000$ K, $R_* = 10\ R_\odot$. Print out each of the following quantities as a function of r_i: $(1-x_i)$, x_i, and $\tau_{\nu_1}(r_i)$, the optical depth at the Lyman limit. Plot these results as a function of r_i.

Part of the point of the problem is that you become familiar with Gaussian quadrature, where one calculates an integral of an arbitrary function by simply evaluating the *integrand* at a few magical points, and adding up the results. That is,

$$\int_0^\infty f(x)e^{-x}dx = \sum_i a_i f(x_i)$$

where a_i are weighting factors for the abcissae x_i.

Print out your nicely commented code.