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# Pole Orientation of Asteroid 44 Nysa via Photometric Astrometry, Including a Discussion of the Method's Application and Its Limitations

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The results of photometric astrometry, a method of determining the orientation of a rotation axis, as applied to asteroid 44 Nysa are presented. The pole orientation of Nysa was found to be $\lambda_0 = 100°$, $\beta_0 = +60°$ with an uncertainty of 10°. The sidereal period is $0^d.26755902 \pm 0.00000006$, and the rotation prograde. Refinements to, and limitations of, the application of the method of photometric astrometry are discussed. In light of the results presented herein, we believe that all photometric astrometry pole determinations of the past should be redone.

## INTRODUCTION

Photometric astrometry is a method for determining the pole orientation, sidereal period, and sense of rotation of solar system objects from measurements of intervals between epochs over long time scales. Details of the method are given in Taylor (1979) and summarized briefly in this paper. We present results of applying this method to asteroid 44 Nysa. Our computer code was checked using Mars as a test case since its sidereal period, sense of rotation, and pole orientation are well known. The discussions on Mars within this paper are given in order to explicate the method of photometric astrometry, its efficacy, and its limitations.

The fundamental input to photometric astrometry is the time that a surface feature on an asteroid, which is normally assumed to be represented by a lightcurve extremum, is observed from the Earth. In the case of Mars, equivalent times are the Universal times of transit of the zero meridian across the center of the geometric disk. These times are tabulated in the *American Ephemeris and Nautical Almanac*. Time shifts (see Section IV of Taylor, 1979) do not need to be applied because the transit times of the zero meridian have already been corrected in a similar fashion.

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## I. OBSERVATIONS

Figure 1 presents five lightcurves of Nysa. The observers and telescopes used are given in the figure caption. Two lightcurves totaling less than 2 hr (from April 12, 1970 and March 30, 1977) are not included. Table I gives the aspect data and photometric parameters for the asteroid. $\lambda$ and $\beta$ are the geocentric longitude and latitude, $\Delta$ and $r$ the geocentric and heliocentric distances, and $\alpha$ the solar phase angle. $V_0(1,\alpha)$ is the magnitude of the primary maximum at phase angle $\alpha$ corrected to unit distance from the Sun and Earth. B−V and U−B are the observed colors. Table II gives positions and photometry data for the comparison stars. Table III presents the lightcurves used in applying photometric astrometry along with an identification label and reference.

## II. ESTIMATING THE SIDEREAL PERIOD

The average of the synodic periods of an asteroid over its entire orbit is called the mean synodic period and defines the num-



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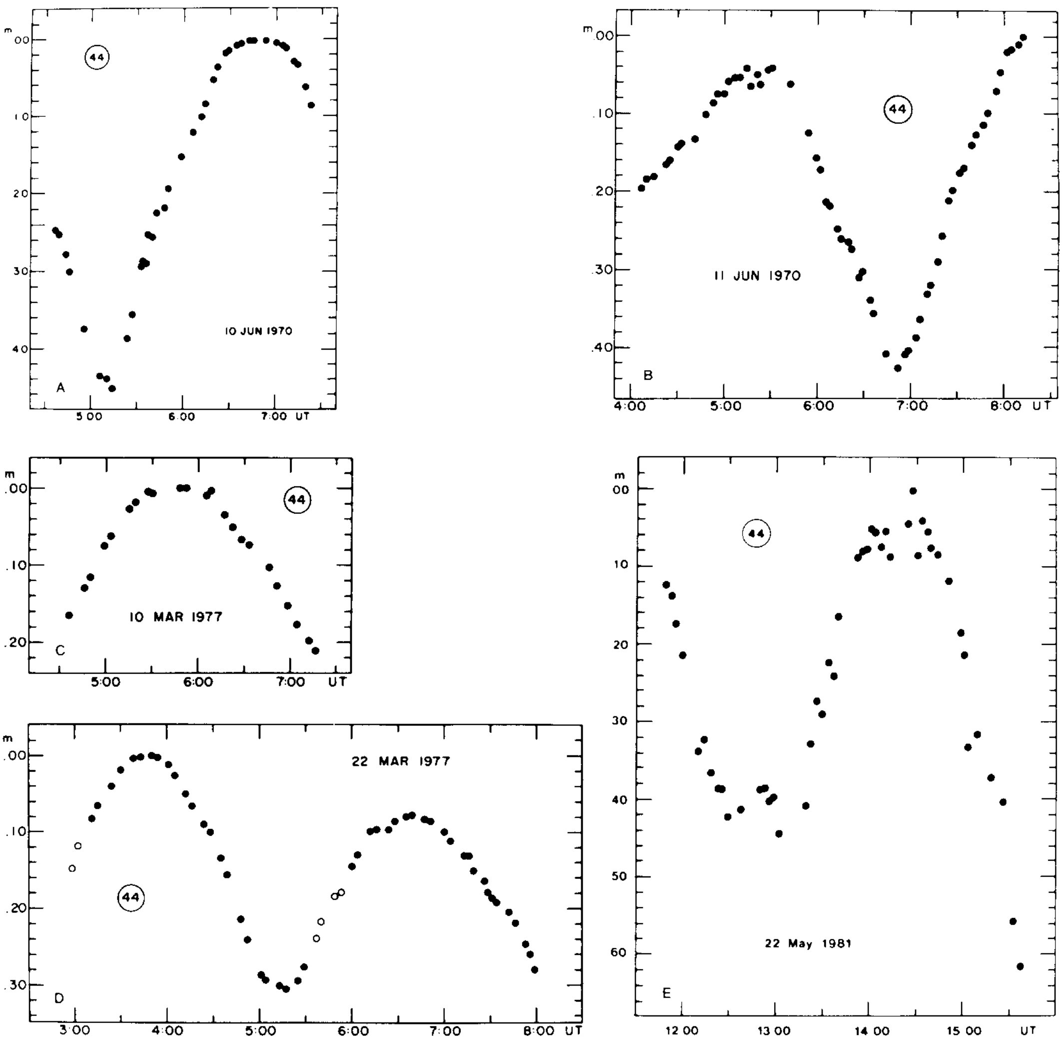


FIG. 1. (A, B) Lightcurves of Nysa obtained by R. E. Sather at the Kitt Peak National Observatory No. 3 41-cm telescope. (C) Lightcurve of Nysa obtained by J. Degewij at the University of Arizona's Catalina Station 102-cm telescope. (D) Lightcurve of Nysa obtained by J. Degewij at the University of Arizona's Catalina Station 155-cm telescope. (E) Lightcurve of Nysa obtained by P. V. Birch at the Perth Observatory 61-cm telescope.

ber of synodic rotation cycles, $N$, over long intervals. Taylor (1979, p. 488) noted that "The most pressing problem in photometric astrometry is . . . the determination of a unique mean synodic period . . ." since this must be known in order to determine $N$.

We can now unambiguously determine the mean synodic period. The mean synodic period will differ from the observed synodic period by only a few seconds and can be found by estimating the asteroid's sidereal rotation period. Each possible mean synodic period generates a different $N$, each of which yields its own pole and sidereal period. The true mean synodic period is the one which most closely agrees with the estimated sidereal period and sense of rotation.

The estimated sidereal period and sense

TABLE I

Aspect Data and Photometry of Nysa

| Date | RA | Declination | $\lambda$ | $\beta$ | $\Delta$ (AU) | $r$ (AU) | $\alpha$ | $V_0(1,\alpha)$ | B–V | U–B |
|---|---|---|---|---|---|---|---|---|---|---|
| | (1950) | | (1950) | | | | | | | |
| 1970 Jun 10 | $14^h\ 4^m.8$ | −6°54′ | 211°.45 | +5°.42 | 1.730 | 2.532 | +17°.1 | 7.39 ± 0.03 | +0.70 ± 0.01 | +0.26 ± 0.01 |
| Jun 11 | 14 4.6 | −6 58 | 211.44 | +5.34 | 1.737 | 2.530 | +17.4 | 7.43 ± 0.03 | +0.69 ± 0.01 | +0.24 ± 0.02 |
| 1977 Mar 10 | 8 36.2 | +20 08 | 126.28 | +1.47 | 1.257 | 1.097 | +18.8 | | | |
| Mar 22 | 8 37.5 | +20 22 | 125.52 | +1.77 | 1.368 | 2.108 | +22.7 | | | |
| 1981 May 22 | 11 30.2 | + 7 55 | 170.00 | +4.30 | 1.776 | 2.313 | +24.4 | | | |

of rotation are found via linear least-squares fit to the object's angular velocity ($\Delta\phi/\Delta t$) on the sky versus synodic periods derived from timings of lightcurve extrema within a single opposition. The intersection at $\Delta\phi/\Delta t = 0$ is the synodic period at the stationary point (i.e., ~ the sidereal period). A positive slope indicates prograde (and a negative slope retrograde) rotation. The absolute value of the slope is $\lesssim (P_{syn})^2/360$ which for Nysa is 0.00019 day$^2$/deg.

Using lightcurve observations from the 1979 apparition (Birch *et al.*, 1982), we computed the 52 independent synodic period–angular velocity pairs given in Table IV and plotted in Fig. 2. Synodic periods derived from superpositions of two lightcurves each having two or more extrema in common were given double weight. A weighted linear least-squares fit to these data gives an estimated sidereal period of $0^d.267581 \pm 0^d.000002$ ($1\sigma$). The slope, $+0.00012 \pm 0.00002$ day$^2$/deg, implies prograde rotation.

## III. MEAN SYNODIC PERIOD

The mean synodic period is obtained by dividing each time interval between pairs of epochs by trial periods until a period is found which gives quotients within ±0.1 of a whole number. There are probably a limitless number of such "mean synodic periods" that could be generated; in this case there were four within a 13-min span. For example, for the Mars data used in our check, the true mean synodic period was $1^d.027497 \pm 0^d.000009$ since it alone gave the estimated sidereal period ($1^d.02596$) and sense of rotation obtained via the analysis discussed in Section II (see Fig. 3) using the techniques reviewed in Section V. About 2 min from the correct mean synodic period lies another at $1^d.028845$. This one, however, gives a sidereal period of $1^d.027305$ (the open circle in Fig. 3) and therefore is rejected. For Mars, we found that for every interval where the difference in solar phase angle was greater than 20° the quotient of the time interval divided by the adopted

TABLE II

IDENTIFICATION AND PHOTOMETRY OF THE COMPARISON STARS AND THE QUALITY OF THE NIGHTS

| Date | RA | Declination (1950) | V | B−V | Scatter of Comparison (mag) | Comments |
|---|---|---|---|---|---|---|
| 1970 Jun 10 | $14^h4^m.7$ | −6°54′ | 11.91 ± 0.01 | +0.96 ± 0.02 | 0.018 | The same star used both nights |
| Jun 11 | 14 4.7 | −6 54 | 11.92 ± 0.03 | +0.94 ± 0.02 | 0.018 | Windy both nights |
| 1977 Mar 10 | Not recorded | | No photometry done | | 0.003 | |
| Mar 22 | Not recorded | | No photometry done | | 0.007 | Four comparison stars used inadvertently |

TABLE III

NYSA LIGHTCURVES USED IN PHOTOMETRIC ASTROMETRY

| Date | ID | Reference |
|---|---|---|
| 1949 Nov 6 | 1 | Shatzel (1954) |
| Nov 7 | 2 | Shatzel (1954) |
| 1954 Jan 6 | 3 | Groeneveld and Kuiper (1954) |
| Jan 7 | 4 | Groeneveld and Kuiper (1954) |
| Jan 11 | 5 | Groeneveld and Kuiper (1954) |
| 1958 Jan 13 | 6 | Gehrels and Owings (1962) |
| 1962 Mar 2 | 7 | Chang and Chang (1962) |
| 1964 Oct 8 | 8 | Yang *et al.* (1965) |
| Oct 29 | 9 | Yang *et al.* (1965) |
| 1970 Jun 10 | 10 | This paper |
| Jun 11 | 11 | This paper |
| 1974 May 16–17 | 12 | Zappalà and van Houten Groeneveld (1979) |
| 1977 Mar 22 | 13 | This paper |
| 1979 Jul 30 | 14 | Birch *et al.* (1983) |
| Jul 31 | 15 | Birch *et al.* (1983) |
| Aug 1 | 16 | Birch *et al.* (1983) |
| Aug 22 | 17 | Birch *et al.* (1983) |
| Aug 24 | 18 | Birch *et al.* (1983) |
| Sep 10 | 19 | Birch *et al.* (1983) |
| Sep 25 | 20 | Birch *et al.* (1983) |
| Sep 26 | 21 | Birch *et al.* (1983) |
| Oct 2 | 22 | Birch *et al.* (1983) |
| Oct 12 | 23 | Birch *et al.* (1983) |
| Oct 17 | 24 | Birch *et al.* (1983) |
| Oct 22 | 25 | Birch *et al.* (1983) |
| Nov 15 | 26 | Birch *et al.* (1983) |
| Dec 7 | 27 | Birch *et al.* (1983) |
| Dec 18 | 28 | Birch *et al.* (1983) |
| 1981 May 22 | 29 | This paper |

mean synodic period differed from a whole number by more than 0.1 and was therefore rejected. This difficulty, although not yet physically understood, is easily avoided by never using intervals involving such large differences in phase angle.

From Table III we selected the Nysa lightcurves (ID = 1–5, 7–11, 16–20, 22, 23, 26) that were used to find its mean synodic period. Lightcurve extrema were used only if their times could be determined to within ±5 min. Intervals were formed from timings between extrema from different oppositions where the solar phase angles differed by less than 20°.

By examining a few test cases it was found that the difference between a mean synodic period and the sidereal period it generated was $\sim 0^d.00005$. Since the estimated sidereal period was $0^d.26758$ (see Section II), it follows that the expected mean synodic period was $\sim 0^d.26763$. A search within a full minute ($0^d.00069$) of this number, using two independent sets of times (lightcurve maxima and minima), gave only one common mean synodic period: $0^d.267611$. Other mean synodic periods exist beyond this 1-min range but result in sidereal periods which differ significantly from the estimated sidereal period. This mean synodic period does indeed generate a sidereal period and sense of rotation which are in agreement with the estimated sidereal period and sense of rotation found in Section II. The two periods differ by only

TABLE IV

DATA FOR ESTIMATING THE SIDEREAL PERIOD OF NYSA

| UT dates of overlays 1979 | | Synodic period | | $\Delta\phi/\Delta t$ | Weight |
|---|---|---|---|---|---|
| Jul 30 | Aug 22 | $0\overset{\mathrm{d}}{.}267571$ ± | $0\overset{\mathrm{d}}{.}000041$ | −.036 | 2 |
| | Aug 24 | 81 | 38 | −.038 | 2 |
| | Sep 10 | 61 | 9 | −.079 | 2 |
| | Sep 25 | 64 | 18 | −.118 | 2 |
| | Sep 26 | 80 | 15 | −.120 | 2 |
| | Oct 2 | 70 | 26 | −.132 | 2 |
| | Oct 12 | 75 | 3 | −.146 | 1 |
| | Oct 17 | 64 | 8 | −.149 | 2 |
| | Oct 22 | 60 | 12 | −.151 | 2 |
| | Nov 15 | 70 | 3 | −.135 | 2 |
| Jul 31 | Aug 22 | 66 | 34 | −.037 | 2 |
| | Aug 24 | 62 | 20 | −.038 | 2 |
| | Sep 10 | 94 | 14 | −.081 | 2 |
| | Sep 25 | 76 | 6 | −.120 | 1 |
| | Sep 26 | 76 | 17 | −.123 | 2 |
| | Oct 17 | 72 | 7 | −.152 | 1 |
| | Oct 22 | 56 | 8 | −.154 | 1 |
| Aug 1 | Aug 22 | 86 | 30 | −.038 | 2 |
| | Aug 24 | 75 | 26 | −.040 | 2 |
| | Sep 10 | 69 | 21 | −.084 | 2 |
| | Sep 25 | 68 | 11 | −.124 | 2 |
| | Sep 26 | 76 | 20 | −.125 | 2 |
| | Oct 17 | 73 | 7 | −.155 | 1 |
| | Oct 22 | 67 | 6 | −.156 | 2 |
| Aug 22 | Sep 10 | 45 | 7 | −.152 | 1 |
| | Sep 25 | 59 | 24 | −.188 | 2 |
| | Oct 2 | 77 | 18 | −.198 | 1 |
| | Oct 12 | 46 | 18 | −.206 | 2 |
| | Oct 17 | 62 | 20 | −.206 | 2 |
| | Oct 22 | 45 | 19 | −.203 | 2 |
| | Nov 15 | 60 | 4 | −.169 | 1 |
| Aug 24 | Sep 10 | 71 | 16 | −.158 | 1 |
| | Sep 25 | 65 | 11 | −.192 | 1 |
| | Sep 26 | 81 | 16 | −.194 | 1 |
| | Oct 12 | 69 | 13 | −.210 | 1 |
| | Oct 22 | 58 | 14 | −.207 | 2 |
| Sep 10 | Sep 25 | 55 | 25 | −.234 | 1 |
| | Sep 26 | 42 | 23 | −.262 | 1 |
| | Oct 2 | 49 | 38 | −.160 | 1 |
| | Oct 12 | 24 | 14 | −.239 | 1 |
| | Oct 17 | 46 | 37 | −.235 | 2 |
| | Oct 22 | 65 | 17 | −.228 | 1 |
| | Nov 15 | 59 | 18 | −.174 | 1 |
| | Dec 18 | 64 | 5 | −.073 | 1 |
| Sep 26 | Oct 22 | 55 | 32 | −.225 | 2 |
| Oct 12 | Nov 15 | 56 | 13 | −.115 | 1 |
| | Dec 18 | 63 | 17 | +.016 | 1 |
| Oct 17 | Nov 15 | 55 | 19 | −.099 | 1 |
| | Dec 7 | 82 | 9 | −.022 | 1 |
| | Dec 18 | 79 | 7 | +.028 | 1 |
| Nov 15 | Dec 7 | 613 | 25 | +.094 | 1 |
| | Dec 18 | $0\overset{\mathrm{d}}{.}267589$ ± | $0\overset{\mathrm{d}}{.}000008$ | +.129 | 1 |

$0\overset{\mathrm{d}}{.}00003$ (see Section V). Table V gives the time interval, $\Delta t$, and number of synodic cycles, $N$, between primary (sharp) minima for pairs of lightcurves from each successive opposition in which this minimum was observed. Note that $N$ is simply the nearest integer to $\Delta t$ divided by the mean synodic period.

## IV. CONSTRAINTS TO APPLYING PHOTOMETRIC ASTROMETRY

Based on the results of applying photometric astrometry to Mars we have reached several conclusions which apply to asteroids.

(1) The final solution from photometric astrometry is independent of the phase angles of the epochs selected. This, however, is not true in determining the mean synodic period from observations obtained within a single opposition (see Section III).

(2) As long as the range in the geocentric ecliptic longitudes of the epochs used is greater than 50°, a solution can be obtained. The application to main belt asteroids is that generally, because of the limited longitude range, the pole cannot be determined from data within just one opposition.

(3) The minimum number of epochs (or for asteroids—high-quality lightcurves) required for a reliable solution is six within one opposition, spaced over as wide an ecliptic longitude span as possible, along with a minimum of one epoch from each of at least four other different oppositions. The six within one opposition are needed to derive the estimated sidereal period (Section II) and the others for both the derivation of the mean synodic period (Section III) and for the final pole determination (Section V). In order to test for possible shifts in the times of extrema, all extrema should be observed each opposition, i.e., a lightcurve covering a complete rotation cycle should be available.

(4) Epochs must be accurately known. The epochs for Mars were altered randomly by up to ±20 min and the pole recalculated. This was repeated 20 times. These random changes ("noise") caused the pole to vary by up to 10°. Hence, if the uncertainty of every epoch is within ±1.5% of the rotation period (±20 min for Mars and ±5.2 min for Nysa), then the pole should be accurate to

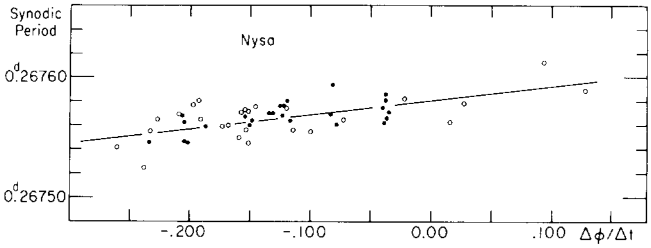


FIG. 2. Nysa: Angular velocity ($\Delta\phi/\Delta t$) versus synodic period from the 1979 lightcurves. Solid symbols are those given double weight (see Section II).

within about 10°. This is the most severe constraint on photometric astrometry. How can one be sure that the time of a lightcurve extremum corresponds to an epoch of astrocentric longitude to such precision? If an asteroid's lightcurves are asymmetric and/or have extrema that shift with respect to time (as is the case for Nysa), then the basic concept of identifying extrema with epochs is questionable. This problem, and its solution are discussed at length in Section V.

## V. APPLYING PHOTOMETRIC ASTROMETRY

The fundamental formula[2] of photometric astrometry is

sideral period

$$= \frac{\Delta T_c}{N \pm (\Delta L/360 + \Delta\delta/360 + \Delta n)}. \quad (1)$$

The numerator is a time interval between two light–time corrected epochs that are an integral number of synodic rotation cycles ($N$) apart. A plus sign following $N$ is used for prograde, and a minus sign for retrograde rotation. The remaining terms of the denominator are corrections to the number of synodic cycles which convert the quotient from a synodic to a sidereal period. These terms are dependent on the pole orientation of the asteroid. $\Delta L/360$ is the fractional part of a cycle that a body would have to rotate in order for the same feature on the surface to be facing the earth at both the beginning and end of the interval. $\Delta\delta/360$ is one-half the fractional part of a cycle that a body would have to rotate in order to move the maximum cross-sectional area from facing the earth to facing the sun. $\Delta n$ is an adjustment of one additional cycle for each orbit about the sun.

Formerly, only intervals of comparable precision were used in Eq. 1. We now use all intervals as follows. The mean of the sidereal periods $\bar{P}$, from all intervals for a given trial pole is $\Sigma\Delta t_i/\Sigma N_i$, where $\Delta t_i$ is the time interval and $N_i$ the number of sidereal cycles—the denominator of Eq. (1). We next define a goodness of fit parameter for

[2] Eq. (1) of Taylor (1979, p. 481) contains a misprint in that the parentheses in the denominator were omitted.

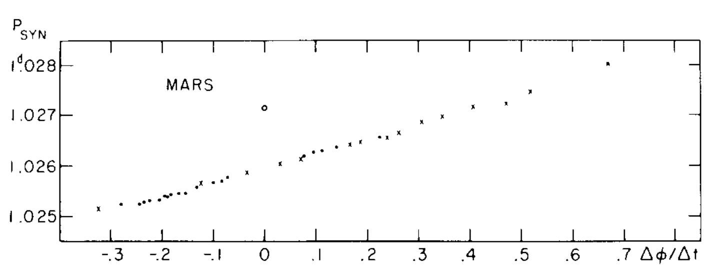


FIG. 3. Mars: Angular velocity ($\Delta\phi/\Delta t$) versus synodic period. From 1971 (●) and from 1975–1976 (x). The open cicle at $\Delta\phi/\Delta t = 0$ is an incorrect period (see Section III for discussion).

TABLE V

NYSA: IDENTIFICATION OF SHARP MINIMA IN THE LIGHTCURVES OF NYSA AND THE NUMBER OF CYCLES OVER LONG INTERVALS

| Date | UT | ID from Table IV | JD(c) | $\Delta t$ | $N$ |
|---|---|---|---|---|---|
| 1949 Nov 6 | 6:06 | 1 | 243 3226$^{\mathrm{d}}$7461 | 1521$^{\mathrm{d}}$9014 | 5687 |
| 1954 Jan 6 | 3:44 | 3 | 4748.6475 | 1468.1254 | 5486 |
| 1958 Jan 13 | 6:42 | 6 | 6216.7729 | 1509.3357 | 5640 |
| 1962 Mar 2 | 14:46 | 7 | 7726.1086 | 951.0692 | 3554 |
| 1964 Oct 8 | 16:28 | 8 | 8677.1778 | 2071.5976 | 7741 |
| 1970 Jun 11 | 6:51 | 11 | 244 0748.7754 | 1435.7361 | 5365 |
| 1974 May 16–17 | 0:30 | 12 | 2184.5115 | 1040.2007 | 3887 |
| 1977 Mar 22 | 5:17 | 13 | 3224.7122 | 918.4285 | 3432 |
| 1979 Sep 26 | 15:35 | 21 | 4143.1407 | 604.0088 | 2257 |
| 1981 May 22 | 15:50 | 29 | 4747.1495 | | |

this trial pole to be $\Sigma(\bar{P} - P_i)N_i/\Sigma N$, where $P_i = \Delta t_i/N_i$. The adopted pole is that which minimizes this parameter.

For the final stage of photometric astrometry on Nysa we used only those lightcurve extrema whose times could be established to within ±5 min. There were too few intervals to obtain a pole using the flat-bottomed minimum since this lightcurve feature appears only when Nysa is observed near equatorial aspect. The other three extrema gave three different pole solutions ($\lambda_0$, $\beta_0$): The maximum preceding the flat-bottomed minimum gave 165°, +35°, the other maximum gave 80°, +35°, while the sharp minimum gave 100°, +60°. Only the last two could even be considered to be real on the basis of their amplitude–aspect relations. We were tempted to adopt the pole derived from the sharp minima simply because their timings were the most precise. However, it was clear that the lightcurves were asymmetric and that the extrema were shifting in time with respect to one another. We therefore had no evidence that the sharp minima (or any other) were fixed and that all the others were shifting.

Given the range in the appearance of Nysa's lightcurves (small amplitude—nearly sine wave to large amplitude—grossly asymmetric) it is not surprising that a unique pole is not obtained when epochs obtained from times of lightcurve extrema observed at greatly differing aspects are used. This is due to the fact that an epoch is defined to be the time of transit of a feature across the center of the observed disk. If the shape of the asteroid is grossly asymmetric then timings of lightcurve extrema observed at different aspects may bear little relation to actual epochs.

The solution to this problem is, however, quite straightforward since for a fixed pole the aspect (the angle between the rotation axis and the earth–asteroid vector) is only a function of the asteroid's geocentric coordinates. One can therefore eliminate the aspect-induced uncertainty in the epochs by including in the analysis only those lightcurves obtained at similar aspects. In practice this means using epochs derived from lightcurves observed at similar ecliptic longitudes (or, in the case of aspects near 90°, similar longitudes or longitudes differing by ~±180°). Since lightcurves observed at the same longitude but different solar phase angles can also appear different, lightcurve features being exaggerated at high phase angles (see Nysa lightcurves in Birch *et al.*, 1983), it is also prudent to restrict the lightcurves used to those obtained at similar phase angles.

When these restrictions were applied to the set of Nysa lightcurves given in Table

III, we were able to form ten intervals using the sharp minima and six using the maximum preceding the minimum. Reducing these separately yielded poles ($\lambda_0$, $\beta_0$) of 93°, +60°(±10°), and 92°, +60°(±10°), respectively. The final results for Nysa's pole orientation were obtained by combining the intervals obtained from these two extrema and adding to them intervals estimated by superposing lightcurves from different oppositions but at similar ecliptic longitudes and solar phase angles.

Table VI gives the intervals, $\Delta t$, and the number of synodic cycles, $N$, used in obtaining the pole orientation for Nysa. The columns $\Delta\alpha$ and $\Delta\lambda$ give the difference in solar phase angle and ecliptic longitude, respectively, for each interval. The penultimate column gives the uncertainty in $\Delta t$ for each superposition of the lightcurves. Note that these uncertainties are never greater than 4 min in ~5500 days, and can be as small as 1 min in 11,000 days. The last column gives the cycle corrections, $\Delta n$, (see Section IV of Taylor, 1979) that were used with the final pole orientation. Two cycle corrections are listed because of the two equivalent north poles, a phenomenon discussed previously by Taylor (1979); there are always two poles (with longitudes differing by ~180° but with similar latitudes) which are solutions to Eq. 1.

The results of applying photometric astrometry to the Nysa data in Table VI are: prograde rotation, a sidereal period of $0^{\mathrm{d}}.26755902 \pm 0^{\mathrm{d}}.00000006$, and a north pole

TABLE VI

SIMILAR ASPECT INTERVALS AND SYNODIC CYCLES FOR NYSA

| Date | UT | ID from Table III | $\Delta t$ | $N$ | $\Delta\alpha$ | $\Delta\lambda$ | Uncertainty | $\Delta n$ |
|---|---|---|---|---|---|---|---|---|
| 1949 Nov 6 | 8:02 | 1 | $5471^{\mathrm{d}}.3107$ | 20445 | $0^{\circ}.7$ | 10° | $\pm 3^{\mathrm{m}}.5$ | 0, −1 |
| 1964 Oct 29 | 15:30 | 9 | | | | | | 4, 3 |
| 1949 Nov 7 | 6:13 | 2 | 5470.2407 | 20441 | 0.3 | 10 | ±1.5 | 0, −1 |
| 1964 Oct 29 | 12:00 | 9 | | | | | | 4, 3 |
| 1949 Nov 6 | 4:30 | 1 | 10937.5319 | 40871 | 0.8 | 20 | ±1.5 | 0, −1 |
| 1979 Oct 17 | 17:17 | 24 | | | | | | 8, 7 |
| 1949 Nov 6 | 7:00 | 1 | 10942.3492 | 40889 | 1.3 | 20 | ±1.0 | 0, −1 |
| 1979 Oct 22 | 15:24 | 25 | | | | | | 8, 7 |
| 1949 Nov 7 | 4:00 | 2 | 10936.4618 | 40867 | 1.2 | 20 | ±2.0 | 0, −1 |
| 1979 Oct 17 | 15:06 | 24 | | | | | | 8, 7 |
| 1949 Nov 7 | 7:00 | 2 | 10941.2790 | 40885 | 0.9 | 20 | ±2.5 | 0, −1 |
| 1979 Oct 22 | 13:43 | 25 | | | | | | 8, 7 |
| 1964 Oct 8 | 17:15 | 8 | 5472.1142 | 20448 | 0.2 | 10 | ±1.0 | 4, 3 |
| 1979 Oct 2 | 20:00 | 22 | | | | | | 8, 7 |
| 1964 Oct 29 | 13:00 | 9 | 5466.2226 | 20426 | 1.5 | 10 | ±2.0 | 4, 3 |
| 1979 Oct 17 | 18:21 | 24 | | | | | | 8, 7 |
| 1964 Oct 29 | 16:00 | 9 | 5471.0412 | 20444 | 0.6 | 10 | ±1.0 | 4, 3 |
| 1979 Oct 22 | 17:00 | 25 | | | | | | 8, 7 |
| 1949 Nov 6 | 8:00 | 1 | 7520.9565 | 28104 | 6.6 | 193 | ±3.0 | 0, −1 |
| 1970 Jun 10 | 7:00 | 10 | | | | | | 5, 5 |
| 1949 Nov 7 | 8:30 | 2 | 7519.8939 | 28100 | 6.2 | 193 | ±3.0 | 0, −1 |
| 1970 Jun 10 | 6:00 | 10 | | | | | | 5, 5 |
| 1949 Nov 6 | 7:13 | 1 | 7522.0307 | 28108 | 6.9 | 193 | ±1.5 | 0, −1 |
| 1970 Jun 11 | 8:00 | 11 | | | | | | 5, 5 |
| 1949 Nov 7 | 7:51 | 2 | 7520.9627 | 28104 | 6.5 | 193 | ±1.5 | 0, −1 |
| 1970 Jun 11 | 7:00 | 11 | | | | | | 5, 5 |
| 1949 Nov 7 | 4:00 | 2 | 11519.3418 | 43045 | 13.5 | 152 | ±3.0 | 0, −1 |
| 1981 May 22 | 12:15 | 29 | | | | | | 8, 8 |

orientation at $\lambda_0 = 100°$ and $\beta_0 = +60°$. (The other north pole is at $\lambda_0 = 265°$ and $\beta_0 = +55°$.)

In order to estimate an uncertainty in the pole orientation, we used the following routine. From Table VI, each of the 14 time intervals was altered by its individual uncertainty, which varied from $\pm 1$ to $\pm 3.5$ min, plus, in a root of the sum of the squares sense, our estimate of the uncertainty in the lightcurve derived epoch, which we estimate to be $\pm 2°$ in rotation phase which corresponds to $\sim \pm 2$ min in Nysa's lightcurves. We then applied these "uncertainties" in a random fashion to the data in Table VI and derived the pole. This was done 20 times. The mean difference in the angle between the adopted pole and these 20 trial poles was 10°.1.

This pole orientation is in agreement with that found by Zappalà and van Houten-Groeneveld (1979): $\lambda_0 = 100° \pm 10°$, $\beta_0 = +50° \pm 10°$. These authors used a magnitude/amplitude–aspect method, a method totally independent of that used in photometric astrometry. The agreement between the derived poles therefore suggests that the poles obtained with both methods are accurate to within their quoted uncertainties. Figure 4 presents the amplitude–aspect plot obtained using the 100°, +60° pole. We define the amplitude to be the difference, in magnitudes, between the primary minimum (listed in Table V) and the following maximum. Note that, due to the high latitude of the pole, the aspect is always within 30° of equatorial. The rather large range in amplitudes at an aspect of 85° is caused by the strong dependence of Nysa's lightcurve amplitudes on solar phase angle (see Birch *et al.*, 1983).

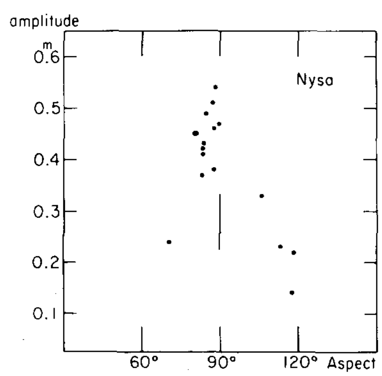


FIG. 4. Amplitude–aspect relation for Nysa.

## VI. CONCLUSIONS AND SUMMARY

The method of photometric astrometry presented in Taylor (1979) need not be changed. The five remaining problems with the application of the method, mentioned in Taylor (1979), have now been solved. These were:

(1) The problem of finding a unique mean synodic period. This is now a straightforward procedure. (See Sections II and III of this paper.)

(2) The problem of inversions, i.e., maxima becoming minima, is resolved. We see no evidence that inversions occur. Nysa's lightcurves can be explained without them. The modeling program of Tedesco and Lambert (1983) has demonstrated this.

(3) Switches, i.e., primary maxima becoming secondary, are real. The primary maxima of the Nysa lightcurves from 1949, 1964, 1970, 1974, 1979, and 1981 switch and become secondary maxima in 1954, 1958, 1962, and 1977. The later group contains observations from the ecliptic longitude range 50 to 145° while the former were observed between 170 and 380°. For Nysa, there do not appear to be any switches involving the minima. $V_0(1,\alpha)$ magnitudes for Nysa from six oppositions are listed in Table VII. Because of switches, the magnitudes of the primary ($M1$) maxima are listed. $M1$, which is not always the brighter of the two maxima, is the maximum that precedes the flat-bottom minimum.

(4) We have demonstrated that the final pole solution is independent of the solar phase angles of the observations but that the mean synodic period derivation is dependent on phase angle differences being less than 20° (Section IV).

(5) We have addressed the problem of the

TABLE VII

PHOTOMETRY FOR NYSA

| UT Date | Year | $V_0(1,\alpha)$ | ID | References |
|---|---|---|---|---|
| Nov 6 | 1949 | 7.35 | M1 | Shatzel (1954) |
| Nov 7 | 1949 | 7.35 | M1 | Shatzel (1954) |
| Jan 6 | 1954 | 7.51 | M2 | Groeneveld and Kuiper (1954) |
| Jan 7 | 1954 | 7.51 | M2 | Groeneveld and Kuiper (1954) |
| Jan 11 | 1954 | 7.51 | M2 | Groeneveld and Kuiper (1954) |
| Jan 13 | 1958 | 7.13 | M2 | Gehrels and Owings (1962) |
| Jun 10 | 1970 | 7.39 | M1 | This paper |
| Jun 11 | 1970 | 7.43 | M1 | This paper |
| May 16–17 | 1974 | 7.13 | M1 | Zappalà and van Houten-Groeneveld (1979) |
| Aug 1 | 1979[a] | 7.57 | M1 | Birch *et al.* (1983) |

[a] See Birch et al. (1983) for a complete listing of the photometry from 15 nights in 1979.

number of lightcurves needed for a photometric astrometry pole determination and the sensitivity of the method to the time of the epochs (Sections IV and V). Having the minimum number of observations given in Section IV does not guarantee a pole solution; the only guarantee is that if the data set is less than these minimum conditions then the pole, if found, will most likely be inaccurate.

The photometric astrometry results for Mars gave a sidereal period within 7 msec of that given in the *American Ephemeris and Nautical Almanac*. The pole orientation agreed to within 1°. We therefore conclude that the accuracy of photometric astrometry is limited only by the precision of the epochs used.

In light of what we have learned in the last 2 years, it is our judgment that it would be wise to redo the pole analyses of the past.

We estimate that photometric astrometry can be applied to six asteroids immediately, namely 4, 5, 6, 16, 20, and 511, and to 10 more after further observations during one apparition. The latter asteroids are numbers 7, 8, 9, 15, 22, 39, 281, 349, 354 and 624.

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